MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2019/2020

PEM0036 – CALCULUS

(Foundation in Engineering)

2 MARCH 2020
2:30 p.m. – 4:30 p.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of **SEVEN (7) pages** including cover page and appendix with **FOUR (4) questions** only.

2. Attempt **ALL FOUR** questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write all your answers in the Answer Booklet provided. All necessary working **MUST** be shown.

4. Only **NON-PROGRAMMABLE** calculator is allowed.

APPENDIX

BASIC DIFFERENTIATION AND INTEGRATION FORMULAS

$$\frac{d}{dx}[\sin x] = \cos x \qquad \frac{d}{dx}[\cos x] = -\sin x$$

$$\frac{d}{dx}[\tan x] = \sec^2 x \qquad \frac{d}{dx}[\cot x] = -\csc^2 x$$

$$\frac{d}{dx}[\sec x] = \sec x \tan x \qquad \frac{d}{dx}[\csc x] = -\csc x \cot x$$

$$\frac{d}{dx}[e^x] = e^x \qquad \frac{d}{dx}[\ln x] = \frac{1}{x}; \quad x > 1$$

$$\frac{d}{dx}[\sin^{-1} x] = \frac{1}{\sqrt{1-x^2}}; \quad |x| < 1 \qquad \frac{d}{dx}[\cos^{-1} x] = -\frac{1}{\sqrt{1-x^2}}; \quad |x| < 1$$

$$\frac{d}{dx}[\tan^{-1} x] = \frac{1}{1+x^2} \qquad \frac{d}{dx}[\cot^{-1} x] = -\frac{1}{1+x^2}$$

$$\frac{d}{dx}[\sec^{-1} x] = \frac{1}{|x|\sqrt{x^2-1}}; \quad |x| > 1 \qquad \frac{d}{dx}[\csc^{-1} x] = -\frac{1}{|x|\sqrt{x^2-1}}; \quad |x| > 1$$

$$Area = \int_a^b [f(x) - g(x)]\ dx$$

$$Volume\ (Disk) = \pi \int_a^b [f(x)]^2\ dx$$

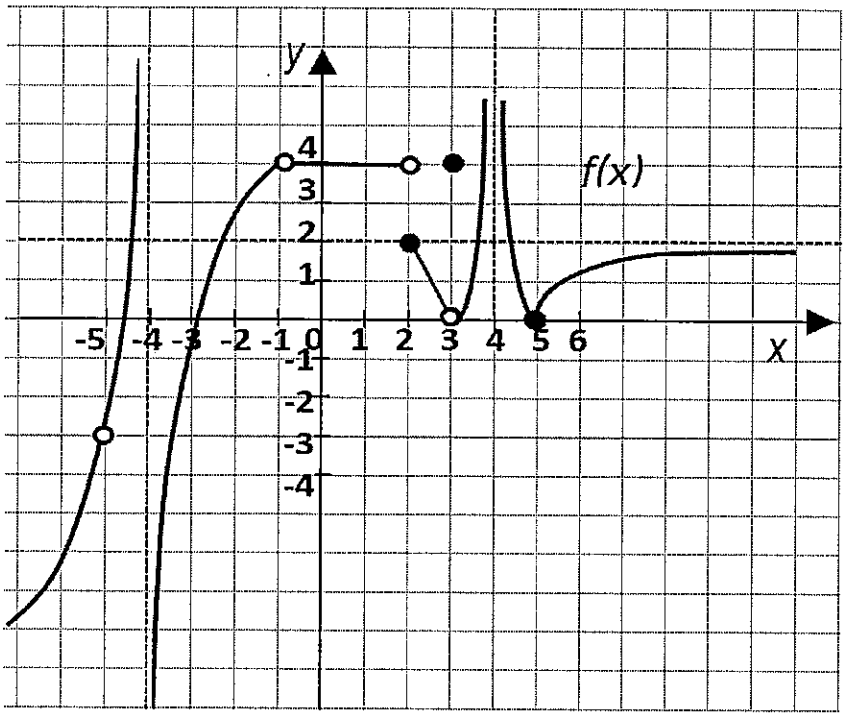
$$Volume\ (Washer) = \pi \int_a^b [f(x)]^2 - [g(x)]^2\ dx$$

$$Volume\ (Cylindrical\ Shells) = \int_a^b 2\pi (shell\ radius)(shell\ height)\ dx$$

Continued…

QUESTION 1 [25 marks]

a. Use the graph $f(x)$ to answer (i) through (xi) below.



Find:

i. $\lim_{x\to -5} f(x)$. [½ mark]

ii. $\lim_{x\to -4} f(x)$. [½ mark]

iii. $\lim_{x\to 2^-} f(x)$. [½ mark]

iv. $\lim_{x\to 2} f(x)$. [½ mark]

v. $\lim_{x\to 3} f(x)$. [½ mark]

vi. $\lim_{x\to 4} f(x)$. [½ mark]

vii. $\lim_{x\to 5} f(x)$. [½ mark]

viii. $\lim_{x\to \infty} f(x)$. [½ mark]

ix. $f(-5)$. [½ mark]

x. $f(5)$. [½ mark]

xi. The x-values, if any, where the function is not continuous. [3 marks]

Continued...

QUESTION 1 [25 marks]

b. Evaluate the following limits:

i. $\lim_{x\to 3^+}\left(\frac{2}{3-x}\right)$ [1 mark]

ii. $\lim_{x\to -\infty}\left(\frac{|x|+1}{2x+3}\right)$ [3 marks]

iii. $\lim_{x\to 0}\left(\frac{e^x-\cos x}{2x}\right)$ [3 marks]

c. Given $f(x)=\frac{10-x}{x^2-100}$.

i. Find the horizontal and vertical asymptotes of $f(x)$. [3 marks]

ii. Find $\lim_{x\to 10} f(x)$. [2 marks]

d. Consider the function $f(x)=\begin{cases}36-x & ; \quad x\neq 6\\ 9+x^2 & ; \quad x=6\end{cases}$.

i. Find $f(6)$. [1½ marks]

ii. Find $\lim_{x\to 6} f(x)$. [1½ marks]

iii. Check whether the function is continuous at $x=6$.
Explain your answer. [2 marks]

Continued...

QUESTION 2 [25 marks]

a. Find the derivative of the functions below with respect to x:

i. $y = 3x^{-2}\left(x^2 + 4\right)$. [2 marks]

ii. $y(\ln x) = \frac{x}{e} - 1$ at $(e, 0)$. [3 marks]

b. Find $f'(x)$ if $f(x) = 5 + \frac{e^{2x} + e^{-x}}{1 + e^x}$. [5 marks]

c. The gradient function of a curve is given by $\lim_{h \to 0} \left(\frac{(x+h)^3 - (x)^3}{h} \right)$.

i. Find the equation of the curve. [1 mark]

ii. Hence, find the equation of the tangent line to this curve at the point where $x = 1$. [4 marks]

d. Given the function $f(x) = 2\cos 2x + 2x$, $0 < x < \pi$. Find the

i. x-coordinates of critical points. [4 marks]

ii. interval on which the function $f(x)$ is increasing and/or decreasing. [3 marks]

iii. local maximum and/or local minimum point(s). [3 marks]

Continued...

QUESTION 3 [25 marks]

a. Integrate the following:

i. $\int (x+2)\sqrt{x^2+4x}\ dx$. [4 marks]

ii. $\int_1^e x \cdot \ln x^4\ dx$. [6 marks]

iii. $\int \left(\frac{x+3}{x^2-x}\right) dx$. [**Hint:** Partial Fraction] [5 marks]

b. Find the shaded area (Figure Q3c) enclosed between the curves $y=-2x+x^2$ and $y=-x+2$ on [-1, 3]. [4 marks]

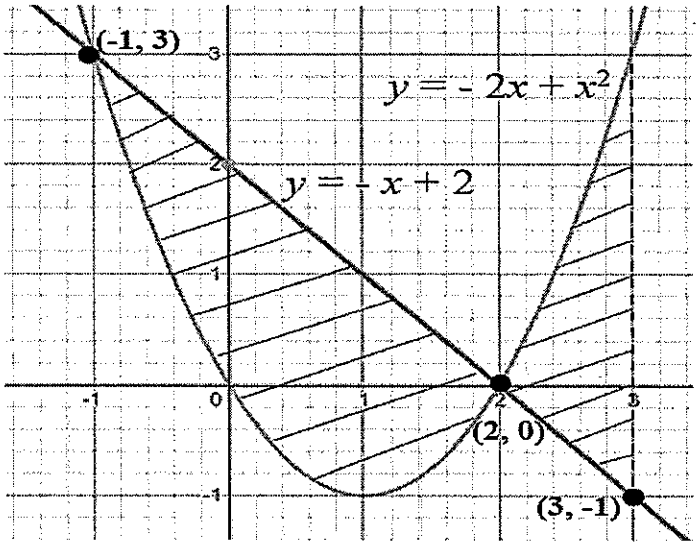


Figure Q3c

c. Consider the functions of $f(x)=|x|$ and $g(x)=5$. Do the following:

i. Sketch the region ***R*** bounded by the graph of $f(x)$ and $g(x)$. Indicate all the intersection points of the bounded region. [2 marks]

ii. The region ***R*** is revolved about the line $y=5$. Find the volume of the resulting solid. [4 marks]

Continued...

QUESTION 4 [25 marks]

a. Find the function that satisfies the following initial value problem.

$$(\ln x)\left(\frac{dy}{dx}\right)=\frac{x\ln x-\ln x}{2} \quad ; \quad y(2)=1. \qquad \text{[4 marks]}$$

b. Solve the following differential equations:

i. $(2x)\left(\frac{dy}{dx}\right)=2x\cdot\cos x-4xy\cdot\cos x.$ [8 marks]

ii. $y''+6y'+9y=e^{2x}-x+1.$ [13 marks]

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